The scientific manuscripts left unpublished by Ettore Majorana

(with outlines of his life and work) [(†)]

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Abstract – We present here a panoramic view of the main *scientific manuscripts* left unpublished by the brightest Italian theoretical physicist of the XX century, Ettore Majorana. We deal in particular: (i) with his very original "study" notes (the so-called *Volumetti*), already published by us in English, in 2003, c/o Kluwer Acad. Press, Dordrecht & Boston, and in the original Italian language, in 2006, c/o Zanichelli pub., Bologna, Italy; and (ii) with a selection of his research notes (the so-called *Quaderni*), that we are going to publish c/o Springer, Berlin. We seize the present opportunity for setting forth also some suitable —scarcely known— information about Majorana's life and work, on the basis of documents (letters, testimonies, various documents...) discovered or collected by ourselves during the last decades. A finished, enlarged version of this paper will appear, as a Preface by the editors, at the beginning of the coming book *Ettore Majorana – Unpublished Research Notes on Theoretical Physics*, edited by S.Esposito, E.Recami, A.van der Merwe and R.Battiston, to be printed by Springer verlag. [Let us recall that almost all the *biographical* documents regarding Ettore Majorana, photos included, are protected by copyright in favor of Maria Majorana together with the present authors (mainly ER), and partly with the publisher Di Renzo (www.direnzo.it), and cannot be further reproduced without the written permission of the right holders].

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1. Ettore Majorana's life and work

"Without listing his works, all of which are highly notable both for their originality of the methods utilized as well as for the importance of the achieved results, we limit ourselves to the following:

"In modern nuclear theories, the contribution made by this researcher to the introduction of the forces called 'Majorana forces' is universally recognized as the one, among the most fundamental, that permits us to theoretically comprehend the reasons for nuclear stability. The work of Majorana today serves as a basis for the most important research in this field.

"In atomic physics, the merit of having resolved some of the most intricate questions on the structure of spectra through simple and elegant considerations of symmetry is due to Majorana.

"Lastly, he devised a brilliant method that permits us to treat the positive and negative electron in a symmetrical way, finally eliminating the necessity to rely on the extremely artificial and unsatisfactory hypothesis of an infinitely large electrical charge diffused in space, a question that had been tackled in vain by many other scholars." [4]

With this justification, the judging committee of the 1937 competition for a new full professorship in theoretical physics at Palermo, chaired by Enrico Fermi (and including Enrico Persico, Giovanni Polvani and Antonio Carrelli) suggested the Italian Minister of National Education to appoint Majorana, "independently of the competition rules, as full-professor of theoretical physics in a university of the Italian kingdom* because of his high and well-deserved reputation" [4]. Evidently, to get such a high reputation there were enough the *few* papers that the Italian scientist had chosen to publish. It is interesting to note that proper light was shed by Fermi on Majorana's symmetrical approach to electrons and anti-electrons (today climaxing in its application to neutrinos and anti-neutrinos) and on its ability in eliminating the hypothesis known as the "Dirac sea": An hypothesis that Fermi defined as "extremely artificial and unsatisfactory," despite the fact that in general it had been uncritically accepted. Though, one of the most important works of Ettore Majorana, the one that introduced his "infinite-components equation," was not mentioned: it had not been yet understood, even by Fermi and colleagues.

Bruno Pontecorvo, a younger colleague of Majorana at the Institute of Physics in

*Which happened to be that of Naples.

Rome, in a similar way recalled that [2]: "some time after his entry into Fermi's group, Majorana already possessed such an erudition and had reached such a high level of comprehension of physics that he was able to speak on the same level with Fermi about scientific problems. Fermi himself held him to be the greatest theoretical physicist of our time. He often was astounded [...]."

Ettore Majorana's fame solidly rests on testimonies like these, and even more on the following ones.

At the request of Edoardo Amaldi [1], Giuseppe Cocconi wrote from CERN (July 18, 1965):

"In January 1938, after having just graduated, I was invited, essentially by you, to come to the Institute of Physics at the University in Rome for six months as a teaching assistant, and once I was there I would have the good fortune of joining Fermi, Gilberto Bernardini (who had been given a chair at Camerino university a few months earlier) and Mario Ageno (he, too, a new graduate), in the research of the products of disintegration of μ "mesons" (at that time called mesotrons or yukons), which are produced by cosmic rays. [...]

"A few months later, while I was still with Fermi in our workshop, news arrived of Ettore Majorana's disappearance in Naples. I remember that Fermi busied himself with telephoning around until, after some days, he had the impression that Ettore would never be found.

"It was then that Fermi, trying to make me understand the significance of this loss, expressed himself in quite a peculiar way; he who was so objectively harsh when judging people. And so, at this point, I would like to repeat his words, just as I can still hear them ringing in my memory: 'Because, you see, in the world there are various categories of scientists: people of a secondary or tertiary standing, who do their best but do not go very far. There are also those of high standing, who come to discoveries of great importance, fundamental for the development of science' (and here I had the impression that he placed himself in that category). 'But then there are geniuses like Galileo and Newton. Well, Ettore was one of them. Majorana had what no one else in the world had [...]'."

Enrico Fermi, who was rather severe in his judgements, again expressed himself in an unusual way on another occasion. On July 27, 1938 (after Majorana's disappearance,

which took place on March 26, 1938), writing from Rome to the Prime Minister Mussolini in order to ask for an intensification of the search for Ettore, he stated:

"I do not hesitate to declare, and it would not be an overstatement in doing so, that of all the Italian and foreign scholars that I have had the chance to meet, Majorana, for his depth of intellect, has struck me the most." [4]

But, nowadays, a part of the interested scholars may find difficult to catch Majorana's ingeniousness by basing themselves only on his few published papers (listed below), most of them originally written in Italian and not easy to be traced; only three articles of his having been translated into English [26,9,10,11,12] in the past. Actually, only in 2006 the Italian Physical Society did eventually publish a book with the Italian and English versions of Majorana's articles [13].

Anyway, the Italian scientist has left also a lot of unpublished manuscripts about his studies and research, mainly deposited at the Domus Galilaeana of Pisa (in Italy), which help illuminating us on his abilities as a theoretical physicist, and mathematician too.

The year 2006 has been the 100th anniversary of the birth of Ettore Majorana: probably the brightest Italian theoretician of the XX century, even if to some people Majorana is known mainly for his mysterious disappearance, in 1938, when he was 31. For celebrating such a centenary, we had been working —among the others— about selection, study, typographical setting in electronic form, and translation into English of the most important research notes left unpublished by Ettore Majorana: His so-called *"Quaderni"* (booklets); leaving aside, for the moment, the noticeable set of loose sheets that constitute a conspicuous part of Majorana's manuscripts. Such a selection is going to be published for the first time, with some understandable delay, in a coming book. In a previous volume [15], that hereafter we shall call *"Volume I"*, we have analogously published for the first time the material contained in *different* Majorana's booklets —the so-called "Volumetti"—, which had been written down by him mainly while *studying* physics and mathematics as an Enrico Fermi's student and collaborator. Even if "Volume I" did already contain many highly original exploits, the preparation of the present book remained nevertheless as a rather necessary enterprize, since the research-notes appearing below are even more (and often exceptionally) interesting, showing at a larger degree Majorana's genius. Many of the following results are still important, and a novelty, for contemporary theoretical physics, even after about eighty years.

2. A further historical prelude

For non specialists, the name of Ettore Majorana is frequently associated with his mysterious disappearance from Naples, on March 26, 1938, when he was only 31; afterwards, in fact, he was never seen again.

But the myth of his "disappearance" [4] has contributed to nothing but the fame he was entitled to, for being a genius well ahead of his time.

Ettore Majorana was born on 5 August 1906 at Catania, Sicily (Italy), to Fabio Majorana and Dorina Corso. The fourth of five sons, he had a rich scientific, technological and political heritage: three of his uncles had become vice-chancellors of the University of Catania and members of the Italian parliament, while another, Quirino Majorana, was a renowned experimental physicist, who had been, by the way, a former president of the Italian Physical Society.

Ettore's father, Fabio, was an engineer who had founded the first telephone company in Sicily and who went on to become chief inspector of the Ministry of Communications. Fabio Majorana was responsible for the education of his son in the first years of his school-life, but afterwards Ettore was sent to study in a boarding school at Rome. Eventually, in 1921, the whole family moved from Catania to Rome. Ettore finished his high-school in 1923 when he was 17, and then joined the Faculty of Engineering of the local university, where he excelled, and counted Giovanni Gentile Jr., Enrico Volterra, Giovanni Enriques, and future Nobel laureate Emilio Segrè among his friends.

In the spring of 1927 Orso Mario Corbino, the director of the Institute of Physics at Rome and an influential politician (who had succeeded in elevating to full-professorship the 25 years old Enrico Fermi, just with the intention to allow the Italian physics to make a quality jump) launched an appeal to the students of Engineering, inviting the most brilliant young minds to study physics. Segrè and Edoardo Amaldi rose to the challenge, joining Fermi and Franco Rasetti's group, and telling them of Ettore's exceptional gifts. After some encouragement from Segrè and Amaldi, Majorana eventually decided to meet Fermi in the autumn of that year.

The details of Majorana and Fermi's first meeting were narrated by E. Segré [3], Rasetti, and Amaldi. The first important work written by Fermi in Rome, on the statistical properties of the atom, is today known as the Thomas-Fermi method. Fermi had found that he needed the solution to a non-linear differential equation characterized by unusual boundary conditions, and in a week of assiduous work he had calculated the solution with a little hand calculator. When Majorana met Fermi for the first time, the

latter spoke about his equation, and showed his numerical results. Majorana, who was always very skeptical, believed Fermi's numerical solution to be probably wrong. He went home, and solved Fermi's original equation in analytic form, evaluating afterwards the solution's values without the aid of any calculator. Next morning he returned to the Institute and skeptically compared the little piece of paper on which he had written his results to Fermi's notebook, and found that their results coincided exactly, he could not hide his amazement; and decided to go on from Engineering to Physics. We have indulged in the foregoing anecdote since the pages on which Majorana solved Fermi's differential equation were found by one of us (S.E.) years ago. And recently [22] it has been explicitly shown that he followed that night two independent paths, the first of them leading to an Abel equation; and the second one devising a method still unknown to mathematics. More precisely, Majorana arrived at a series solution of the Thomas-Fermi equation by using an original method that applies to an entire class of mathematical problems. More in general, while some of Majorana's results anticipated by several years those of renowned mathematicians or physicists, several others (including his final solution to the mentioned equation) have not been obtained by anyone else since: Such facts are further evidence of Majorana's brilliance.

3. Majorana's published articles

Majorana published few scientific articles: nine, actually, besides his sociology paper entitled "Il valore delle leggi statistiche nella fisica e nelle scienze sociali" (The value of statistical laws in physics and the social sciences), which was however published not by Majorana but (posthumously) by G. Gentile Jr., in *Scientia* [**36** (1942) 55-56], and much later translated into English. Majorana switched from the Engineering to the Physics studies in 1928 (the year in which he published his first article, written in collaboration with his friend Gentile) and then went on to publish his works in theoretical physics only for few years, practically only until 1933. Nevertheless, even his *published* works are a mine of ideas and techniques of theoretical physics that still remains largely unexplored. Let us list his nine published articles, which only in 2006 have been eventually reprinted together with their English translation [13]:

(1) "Sullo sdoppiamento dei termini Roentgen ottici a causa dell'elettrone rotante e

sulla intensità delle righe del Cesio," in collaboration with Giovanni Gentile Jr., *Rendiconti Accademia Lincei* **8** (1928) 229-233.

(2) "Sulla formazione dello ione molecolare di He," *Nuovo Cimento* **8** (1931) 22-28.

(3) "I presunti termini anomali dell'Elio," *Nuovo Cimento* **8** (1931) 78-83.

(4) "Reazione pseudopolare fra atomi di Idrogeno," *Rendiconti Accademia Lincei* **13** (1931) 58-61.

(5) "Teoria dei tripletti *P'* incompleti," *Nuovo Cimento* **8** (1931) 107-113.

(6) "Atomi orientati in campo magnetico variabile," *Nuovo Cimento* **9** (1932) 43-50.

(7) "Teoria relativistica di particelle con momento intrinseco arbitrario," *Nuovo Cimento* **9** (1932) 335-344.

(8) "Über die Kerntheorie," *Zeitschrift für Physik* **82** (1933) 137-145; "Sulla teoria dei nuclei," *La Ricerca Scientifica* **4**(1) (1933) 559-565.

(9) "Teoria simmetrica dell'elettrone e del positrone," *Nuovo Cimento* **14** (1937) 171-184.

While still an undergraduate, then, in 1928 Majorana published his first paper, (1), in which he calculated the splitting of certain spectroscopic terms in gadolinium, uranium and caesium, due to the spin of the electrons. At the end of that same year, Fermi invited Majorana to give a talk at the Italian Physical Society on some applications of the Thomas-Fermi model [23] (attention to which has been called by F. Guerra and N. Robotti) . Then on 6 July 1929, Majorana got his master degree in physics, with a dissertation having as a subject "The quantum theory of radioactive nuclei."

By the end of 1931 the 25-year-old physicist had published two articles, (2) and (4), on the chemical bond of molecules, and two more papers, (3) and (5) on spectroscopy, one of which, (3), anticipated results later obtained by a Samuel Goudsmith's collaborator on the "Auger effect" in helium. As Edoardo Amaldi has written, an in-depth examination of these works leaves one struck by their quality: They reveal both a deep knowledge of the experimental data, even in the minutest detail, and an uncommon ease, without equal at that time, in the use of the symmetry properties of the quantum states in order to

qualitatively simplify problems and choose the most suitable method for their quantitative resolution.

In 1932, Majorana published an important paper, (6), on the non-adiabatic spin-flip of atoms in a magnetic field, which was later extended by Nobel laureate Rabi in 1937, and by Bloch and Rabi in 1945. It established the theoretical basis for the experimental method used to reverse the spin also of neutrons by a radio-frequency field, a method that is still practiced today, for example, in all polarized-neutron spectrometers. That paper contained an independent derivation of the well-known Landau-Zener formula (1932) for non-adiabatic transition probability. It also introduced a novel mathematical tool for representing spherical functions or, rather, for representing spinors by a set of points on the surface of a sphere (Majorana sphere), attention to which has been called not long ago by R. Penrose and collaborators [27] (and by C.Leonardi and coworkers [28]). In the *present* Volume the reader will find some additions (or modifications) to the above-mentioned published articles.

However, the most important 1932 paper is that concerning a relativistic field theory of particles with arbitrary spin, (7). Around 1932 it was commonly believed that one could write relativistic quantum equations only in the case of particles with spin zero or one half. Convinced of the contrary, Majorana —as we knew since long time from his manuscripts, constituting a part of the *Quaderni* finally published here— began constructing suitable quantum-relativistic equations for higher spin values (one, three-halves, etc.); and he even devised a method for writing the equation for a generic spin-value. But still he published nothing[†], until he discovered that one could write a single equation to cover an infinite family of particles of arbitrary spin (even if at that time the known particles could be counted on one hand). In order to implement his programme with these "infinite components" equations, Majorana invented a technique for the representation of a group several years before Eugene Wigner did. And, what is more, Majorana obtained the infinite-dimensional unitary representations of the Lorentz group that will be re-discovered by Wigner in his 1939 and 1948 works. The entire theory was re-invented by Soviet series of articles from 1948 to 1958, and finally applied by physicists years later. Sadly, Majorana's initial article remained in the shadows for a good 34 years until D. Fradkin, informed by E. Amaldi, realized what Majorana many years earlier had accomplished [26]. All the scientific material contained in (and in preparation of) this Majorana's publication is illuminated by the manuscripts constituting Majorana's Quaderni, a selection of which

[†]Starting with 1974, some of us [21] published and revaluated only few of the pages devoted in its manuscripts by Majorana to the case of a Dirac-like equation for the photon (spin-1 case).

—as already mentioned— will appear in a book [35] that we shall call *"Volume II"*.

At the beginning of 1932, as soon as the news of the Joliot-Curie experiments reached Rome, Majorana understood that they had discovered the "neutral proton" without having realized it. Thus, even before the official announcement of the discovery of the neutron, made soon afterwards by Chadwick, Majorana was able to explain the structure and stability of light atomic nuclei with the help of protons and neutrons, antedating in this way also the pioneering work of D. Ivanenko, as both Segré and Amaldi have recounted. Majorana's colleagues remember that even before Easter he had concluded that protons and neutrons (indistinguishable with respect to the nuclear interaction) were bound by the "exchange forces" originating from the exchange of their spatial positions alone (and not also of their spins, as Heisenberg would propose), so as to produce the alpha particle (and not the deuteron) as saturated with respect to the binding energy. Only after Heisenberg had published his own article on the same problem was Fermi able to persuade Majorana to go for a six-months period, in 1933, to Leipzig, and meet there his famous colleague (who will be awarded the Nobel price at the end of that year); and finally Heisenberg was able to convince Majorana to publish his results in the paper "Über die Kerntheorie." Actually, Heisenberg had interpreted the nuclear forces in terms of nucleons exchanging spinless electrons, as if the neutron were formed in practice by a proton and an electron; whilst Majorana had simply considered the neutron as a "neutral proton": and the theoretical and experimental consequences were quickly recognized by Heisenberg. Majorana's paper on the stability of nuclei became soon known to the scientific community —a rare event, as we know, for his writings— thanks to that timely "propaganda" made by Heisenberg himself, who on several occasions, when discussing the "Heisenberg-Majorana" exchange forces, used, rather fairly and generously, to point out more Majorana's than his own contributions [31]. The manuscripts published in the present book refer also to what Majorana wrote down before having read Heisenberg's paper. Let us seize the present opportunity to quote two brief passages from Majorana's letters from Leipzig. On February 14, 1933, he writes his mother (the italics are ours): "The environment of the physics institute is very nice. I have good relations with Heisenberg, with Hund, and with everyone else. *I am writing some articles in German. The first one is already ready* [...]" [4]. The work that was already ready is, naturally, the cited one on nuclear forces, which, however, remained *the only paper* in German. Again, in a letter dated February 18, he tells his father (we italicize): "*I will publish in German, after having extended it, also my latest article which appeared in Il Nuovo Cimento*" [4].

But Majorana published nothing more, either in Germany —where he had become

acquainted, besides with Heisenberg, with other personalities, including Ehrenfest, Bohr, Weisskopf, Bloch— or after his return to Italy: Except for the article (in 1937) of which we are about to speak. It is therefore of importance to know that Majorana was engaged in writing other papers: in particular, he was expanding his article about the infinite-components equations. His research activity during the years 1933-37 is testified by documents presented below, and particularly by a number of Majorana's unpublished scientific notes, part of which are reproduced here: As far as we know, it was focused mainly on field theory and quantum electrodynamics. As we already mentioned, in 1937 Majorana decided to compete, however, for a full-professorship (probably with the only desire to have students); and he was urged to demonstrate that he was still actively working in theoretical physics: Happily enough, then, he took from a drawer[‡] his writing on the symmetric theory of electrons and anti-electrons, publishing it that same year under the title "Symmetric theory of electron and positron." This paper —at present, the most famous of his, probably— was initially noticed almost exclusively for having introduced the Majorana representation of the Dirac matrices in real form. But its main consequence is that a neutral fermion can be identical with its anti-particle. Let us stress that such a theory was rather revolutionary, since it was at variance with what Dirac had successfully assumed in order to solve the problem of negative energy states in quantum field theory. With rare daring, Majorana suggested that neutrinos, which had just been postulated by Pauli and Fermi to explain puzzling features of radioactive beta decay, could be particles of this type. This would enable the neutrino, for instance, to have mass; which may have a bearing on the phenomena of neutrino oscillations, later postulated by Pontecorvo.

It may be stressed that, exactly as in the case of other writings of his, the "Majorana neutrino" too started to gain prominence only decades later, beginning in the 1950s; and nowadays expressions like Majorana spinors, Majorana mass, and even "majorons", are fashionable: It is moreover well-known that many experiments are currently devoted all the world over to check whether the neutrinos are of the Dirac or the Majorana type. We have already said that the material itself, *published* by Majorana (but still little known, despite it all), constitutes a potential gold-mine for physics. Many years ago, for example, Bruno Touschek noticed that the present article implicitly contains also what he called the theory of the "Majorana oscillator," described by the simple equation $q + \omega^2 q = \varepsilon\ \delta(t)$, where ε is a constant and δ the Dirac function [4]. According to Touschek, the properties of the Majorana oscillator are very interesting, especially in connection with its energy

[‡]As we said, from the existing manuscripts it appears that Majorana had formulated also the essential lines of his paper (9) during the years 1932-33.

spectrum: But no literature seems to exist yet on it.

4. An account of the unpublished manuscripts

The largest part of the Majorana's work was left unpublished. Even if the most important manuscripts have gone probably lost, we are now in possession of: (i) his M.Sc thesis on "The quantum theory of radioactive nuclei"; (ii) 5 notebooks (the *Volumetti*); 18 booklets (the *Quaderni*); (iii) 12 folders with loose papers, and (iv) the set of his lecture notes for the course on theoretical physics given by him at the University of Naples. With the collaboration of E. Amaldi, all these manuscripts were deposited by Luciano Majorana (Ettore's brother) at the Domus Galilaeana of Pisa, Italy. An analysis of those manuscripts allowed us to ascertain that they, except for the lectures notes, appear to have been written approximately by 1933 (even the essentials of his last article, which Majorana proceeded to publish, as we already know, in 1937, seem to have been ready by 1933, the year in which the discovery of the positron was confirmed). Besides the material deposited at the Domus Galilaeana, we have been in possession of: (v) a series of 34 letters written by Majorana between March 17, 1931, and November 16, 1937, in reply to his uncle Quirino —a renowned experimental physicist and a former president of the Italian Physical Society— who had been pressing Majorana for help in the theoretical explanation of his experiments:[§] such letters have been recently deposited at the Bologna University, and have been published in their entirety by G. Dragoni [8]. They confirm that Majorana was deeply knowledgeable even about experimental details. Moreover, Ettore's sister, Maria, recalled that, even in those years, Majorana —who had reduced his visits to Fermi's Institute, starting from the beginning of 1934 (that is, just after his return from Leipzig)— continued to study and work at home many hours during the day and at night. Did he continue to dedicate himself to physics? From one of those letters of his to Quirino, dated January 16, 1936, we find a first answer, because we get to learn that Majorana had been occupied "since some time, with quantum electrodynamics;" knowing Majorana's love for understatements, this no doubt means that during 1935 Majorana had performed profound research at least in the field of quantum electrodynamics.

This seems to be confirmed by a recently retrieved text, written by Majorana in French [25], where the author dealt with a peculiar topic in quantum electrodynamics. It

[§]In the past, one of us (E.R.) have been able to publish only short passages of them, since they are rather technical: cf. Ref.[4].

is instructive, as to that topic, just to quote directly from the Majorana's paper:

≪Let us consider a system of p electrons and set the following assumptions: 1) the interaction between the particles be sufficiently small, allowing to speak about individual quantum states, so that one may regard the quantum numbers defining the configuration of the system as good quantum numbers; 2) any electron has a number $n > p$ of inner energetic levels, while any other level has a much greater energy. One deduces that the states of the system as a whole may be divided into two classes. The first one is composed of those configurations for which all the electrons belong to one of the inner states. Instead, the second one is formed by those configurations in which at least one electron belongs to a higher level not included in the above-mentioned n levels. We shall also assume that it is possible, with a sufficient degree of approximation, to neglect the interaction between the states of the two classes. In other words, we will neglect the matrix elements of the energy corresponding to the coupling of different classes, so that we may consider the motion of the p particles, in the n inner states, as if only these states existed. Our aim becomes, then, translating this problem into that of the motion of $n - p$ particles in the same states, such new particles representing the holes, according to the Pauli principle.≫

Majorana, thus, applied the formalism of field quantization to Dirac's hole theory, obtaining a general expression for the QED Hamiltonian in terms of anticommuting "hole quantities". Let us point out that, in justifying the use of anticommutators for fermionic variables, Majorana commented that such a use "cannot be justified on general grounds, but only by the particular form of the Hamiltonian. In fact, we may verify that the equations of motion are better satisfied by these relations than by the Heisenberg ones." In the second (and third) part of the same manuscript, Majorana took into consideration also a reformulation of QED in terms of a photon wavefunction: a topic that was particularly studied in his *Quaderni* (and is reproduced here). Majorana, indeed, reformulated quantum electrodynamics by introducing a real-valued wave function for the photon, corresponding only to directly observable degrees of freedom.

In some other manuscripts, probably prepared for a seminar at the Naples University in 1938 [24], Majorana set forth a physical interpretation of quantum mechanics, that anticipated of several years the Feynman approach in terms of path integrals. The starting point in Majorana's notes was to search for a meaningful and clear formulation of the concept of quantum state. Afterwards, the crucial point in the Feynman formulation of quantum mechanics (namely, that of considering not only the paths corresponding to

classical trajectories, but all the possible paths joining an initial point with the final point) was really introduced by Majorana, after a discussion about an interesting example of harmonic oscillator. Let us also emphasize the key role played by the symmetry properties of the physical system in the Majorana analysis; a feature quite common in this author's papers.

Do any other unpublished scientific manuscripts of Majorana exist? The question, raised by his answer to Quirino and by his letters from Leipzig to his family, becomes of greater importance when one reads also his letters addressed to the National Research Council of Italy (CNR) during that period. In the first one (dated January 21, 1933), Majorana asserts: "At the moment, I am occupied with the elaboration of a theory for the description of arbitrary-spin particles that I began in Italy and of which I gave a summary notice in Il Nuovo Cimento [...]" [4]. In the second one (dated March 3, 1933) he even declares, referring to the same work: "I have sent an article on nuclear theory to Zeitschrift für Physik. I have the manuscript of a new theory on elementary particles ready, and will send it to the same journal in a few days" [4]. Considering that the article described above as a "summary notice" of a new theory was already of a very high level, one can imagine how interesting it would be to discover a copy of its final version, which went unpublished. [Is it still, perhaps, in the Zeitschrift für Physik archives? Our search ended till now in failure].

A few of Majorana's other ideas, which did not remain concealed in his own mind, have survived in the memories of his colleagues. One such reminiscence we owe to Gian Carlo Wick. Writing from Pisa on October 16, 1978, he recalls: "The scientific contact [between Ettore and me], mentioned by Segré, happened in Rome on the occasion of the 'A. Volta Congress' (long before Majorana's sojourn in Leipzig). The conversation took place in Heitler's company at a restaurant, and therefore without a blackboard [...]; but even in the absence of details, what Majorana described in words was a 'relativistic theory of charged particles of zero spin based on the idea of field quantization' (second quantization). When much later I saw Pauli and Weisskopf's article [*Helv. Phys. Acta* **7** (1934) 709], I remained absolutely convinced that what Majorana had discussed was the same thing [...]" [4].

5. Teaching theoretical physics

As we have seen, Majorana did significantly contribute to theoretical research which was among the frontier topics in the 1930s, and, indeed, in the following decades. However, he deeply thought also on the basics, and applications, of quantum mechanics: and Majorana's lectures on theoretical physics forward an evidence of this work of his.

As realized only recently [32], Majorana revealed a genuine interest in advanced physics teaching, starting from 1933, just after he obtained, at the end of 1932, the degree of "libero docente" (analogous to the German "privatdozent" title). As allowed by that degree, he requested to be allowed to give three subsequent annual free courses at the University of Rome, between 1933 and 1937, as testified by the lecture programs proposed by him and still present in the Rome University's archives. Such documents too refer to a period of time that had been regarded by his colleagues as Majorana's "gloomy years". Although it seems that Majorana never delivered such three courses, probably due to lack of appropriate students, the chosen topics to be lectured appear very interesting and informative.

The first course (academic year 1933-34) proposed by Majorana was that of Mathematical Methods of Quantum Mechanics.[¶]

The second proposed course (academic year 1935-36) was instead that of Mathematical Methods of Atomic Physics.[‖]

Finally, the requested third course (academic year 1936-37) was that of Quantum Electrodynamics.[**]

[¶] The program for it contained the following topics: (i) Unitary geometry. Linear transformations. Hermitian operators. Unitary transformations. Eigenvalues and eigenvectors; (ii) Phase space and the quantum of action. Modifications of classical kinematics. General framework of quantum mechanics; (iii) Hamiltonians which are invariant under a transformation group. Transformations as complex quantities. Non compatible systems. Representations of finite or continuous groups; (iv) General elements on abstract groups. Representation theorems. The group of spatial rotations. Symmetric groups of permutations and other finite groups; (v) Properties of the systems endowed with spherical symmetry. Orbital and intrinsic momenta. Theory of the rigid rotator; (vi) Systems with identical particles. Fermi and Bose-Einstein statistics. Symmetries of the eigenfunctions in the center-of-mass frames; (vii) Lorentz group and spinor calculus. Applications to the relativistic theory of the elementary particles.

[‖] The corresponding subjects were: Matrix calculus. Phase space and the correspondence principle. Minimal statistical sets or elementary cells. Elements of quantum dynamics. Statistical theories. General definition of symmetry problems. Representations of groups. Complex atomic spectra. Kinematics of the rigid body. Diatomic and polyatomic molecules. Relativistic theory of the electron and the foundations of electrodynamics. Hyperfine structures and alternating bands. Elements of nuclear physics.

[**] The main topics were: Relativistic theory of the electron. Quantization procedures. Field quantities defined by commutability and anticommutability laws. Their kinematical equivalence with sets with an

Majorana could actually lecture on theoretical physics only in 1938 when, as recalled above, he obtained his position as a full-professor in Naples. He gave his lectures starting on January 13 and ending with his disappearance (March 26), but his activity was intense, and his interest for teaching very high. For the benefit of his students, and perhaps also for writing down a book, he prepared careful lecture notes [17,18]. A recent analysis [34] showed that Majorana's 1938 course was very innovative for that time, and this has been confirmed by the retrieval (on September 2004) of a faithful transcription of the whole set of Majorana's lecture notes (the so-called "Moreno document") comprising the 6 lectures not included in the original collection [19].

The first part of his course on theoretical physics dealt with the phenomenology of the atomic physics and its interpretation in the framework of the old Bohr-Sommerfeld quantum theory. This part presents a strict analogy with the course given by Fermi in Rome (1927-28), attended by Majorana when a student. The second part started, instead, with the classical radiation theory, reporting explicit solutions to the Maxwell equations, scattering of the solar light and some other applications. It then continued with the theory of Relativity: after the presentation of the corresponding phenomenology, a complete discussion of the mathematical formalism required by that theory was given, ending with some applications as the relativistic dynamics of the electron. Then, it followed a discussion of important effects for the interpretation of quantum mechanics, such as photoelectric effect, Thomson scattering, Compton effects and the Franck-Hertz experiment. The last part of the course, more mathematical in nature, treated explicitly quantum mechanics, both in the Schrödinger and in the Heisenberg formulations. This part did not follow the Fermi approach, but rather referred to personal previous studies, getting also inspiration from Weyl's book on group theory and quantum mechanics.

6. A brief sketch of "Volume I" (which includes Majorana's "study" notes)

In Volume I we have reproduced, and translated, Majorana's *Volumetti*: that is, his *study* notes, written in Rome between 1927 and 1932. Each one of those neatly organized book-

undetermined number of objects obeying the Bose-Einstein or Fermi statistics, respectively. Dynamical equivalence. Quantization of the Maxwell-Dirac equations. Study of the relativistic invariance. The positive electron and the symmetry of charges. Several applications of the theory. Radiation and scattering processes. Creation and annihilation of opposite charges. Collisions of fast electrons.

lets, prefaced by a table of contents, consisted in about 100−150 sequentially numbered pages, while a date, penned on its first blank, recorded the approximate time during which it was completed. Each *Volumetto* was written during a period of about one year. The contents of those notebooks range from typical topics covered in academic courses to topics at the frontiers of research: despite this unevenness in the level of sophistication, the style is never obvious. As an example, we can recall Majorana's study of the shift in the melting point of a substance when it is placed in a magnetic field, or his examination of heat propagation using the "cricket simile." As to frontier research arguments, we can recall two examples: the study of quasi-stationary states, anticipating Fano's theory, and the already mentioned Fermi's theory of atoms, reporting analytic solutions of the Thomas-Fermi equation with appropriate boundary conditions in terms of simple quadratures. He also treated therein, in a lucid and original manner, contemporary physics topics such as Fermi's explanation of the electromagnetic mass of the electron, the Dirac equation with its applications, and the Lorentz group.

Just to give a very short account of the interesting material present in the *Volumetti*, let us point out the following.

First of all, we already mentioned that in 1928, when Majorana was starting to collaborate (still as a University student) with the Fermi group in Rome, he already revealed his outstanding ability in solving involved mathematical problems in original and clear ways, by obtaining an analytical series-solution of the Thomas-Fermi equation. Let us recall once more that his whole work on this topic was written in some loose sheets, and then diligently transcribed by the author himself in his *Volumetti*: so that it is contained in Volume I. From those pages, it appears evident also the contribution given by Majorana to the relevant statistical model, anticipating some important results found later on by leading specialists. As to the major Majorana's finding (namely, his methods of solutions of that equation), let us stress that it remained completely unknown until very recent times, at the extent that the physicist community ignored that the non-linear differential equations, relevant for atoms and for other systems too, can be solved semi-analytically (see Sect. 7 of *Volumetto II*). Indeed, a noticeable property of the method invented by Majorana for solving the Thomas-Fermi equation, is that it may be easily generalized, and may then be applied to a large class of particular differential equations. Several generalizations of his method for atoms were proposed by Majorana himself: they have been rediscovered only many years later. For example, in Sect. 16 of *Volumetto II*, Majorana studied the problem of an atom in a weak external electric field, that is, the problem of atomic polarizability, and obtained an expression for the electric dipole

moment for a (neutral or arbitrarily ionized) atom. Furthermore, he also started applying the statistical method to molecules, rather than single atoms, by studying the case of a diatomic molecule with identical nuclei (see Sect. 12 of *Volumetto II*). Finally, our author considered the second approximation for the potential inside the atom, beyond the Thomas-Fermi approximation, by generalizing the statistical model of neutral atoms to those ionized n times, the case $n = 0$ included (see Sect. 15 of *Volumetto II*). As recently pointed out by one of us (S.E.) [23], the approach used by Majorana to this end is rather similar to the one now adopted in the renormalization of physical quantities, in modern gauge theories.

As well documented, Majorana was among the first ones to study nuclear physics in Rome (we already know that in 1929 he defended a M.Sc. thesis on such a subject). But he continued to do research on similar topics for several years, till his famous 1933 theory of nuclear exchange-forces. For (α, p) reactions on light nuclei, whose experimental results had been interpreted by Chadwick and Gamov, in 1930 Majorana elaborated a dynamical theory (in Sect. 28 of *Volumetto IV*) by describing the energy states associated with the superposition of a continuous spectrum and one discrete level [33]. Actually, Majorana provided a complete theory for the artificial disintegration of nuclei bombarded by α particles (with and without α absorption). He approached this question by considering the simplest case, with a single unstable state of a nucleus and an α particle, which spontaneously decays by emitting an α particle or a proton. The explicit expression for the total cross-section was also given, rendering his approach accessible to experimental checks. Let us emphasize that the peculiarity of Majorana's theory was the introduction of quasi-stationary states, which were considered by U. Fano in 1935 (in a quite different context), and widely used in condensed matter physics about 20 years later.

In Sect. 30 of *Volumetto II*, Majorana made an attempt to find a relation between the fundamental constants e, h, c. The interest of this work resides less in the particular mechanical model adopted by Majorana (which led, indeed, to the result $e^2 \simeq hc$ far from the true value, as noticed by the author himself), than in the adopted interpretation of the electromagnetic interaction, in terms of particle exchange. Namely, the space around charged particles was regarded as quantized, and electrons interacted by exchanging particles; Majorana's interpretation substantially coincides with that introduced by Feynman in quantum electrodynamics after more than a decade, when the space surrounding charged particles will be identified with the QED vacuum, while the exchanged particles will be assumed to be photons.

Finally, one cannot forget the pages contained in *Volumetti III* and *V* on group theory,

where Majorana showed in detail the relationship existing between the representations of the Lorentz group and the matrices of the (special) unitary group in two dimensions. In those pages, aimed also at extending Dirac's approach, Majorana deduced the *explicit* form of the transformations of every bilinear quantities in the spinor fields. Certainly, the most important result achieved by Majorana on this subject is his discovery of the *infinite-dimensional* unitary representations of the Lorentz group: He set forth the *explicit* form of them too (see Sect. 8 of *Volumetto V*, besides his published article (7).) We already recalled that such representations were rediscovered by Wigner only in 1939 and 1948, and later on, in the years 1948-1958, eventually studied by many authors. People like van der Waerden recognized the importance, also mathematical, of such a Majorana's result, but, as we know, it remained unnoticed till the above-quoted 1966 Fradkin's article.

7. About Majorana's research notes ("Volume II")

The material reproduced in Volume I was a paragon of order, conciseness, essentiality and originality. So much so that those notebooks can be partially regarded as an innovative text of theoretical physics, even after about eighty years; besides being another gold-mine of theoretical, physical, and mathematical ideas and hints, stimulating and useful for modern research too.

But Majorana's most remarkable scientific manuscripts —namely, his *research* notes— are represented by a host of loose papers and by the *Quaderni*: and what we called "Volume II" will reproduce[35] a selection of the latter. But the manuscripts with Majorana's research notes, at variance with the *Volumetti*, do rarely contain any introductions or verbal explanations.

The topics faced in the *Quaderni* range from classical physics to quantum field theory, and comprise the study of a number of applications for atomic, molecular and nuclear physics. Particular attention was reserved to the Dirac theory and its generalizations, and to quantum electrodynamics.

The Dirac equation describing spin-1/2 particles was mostly considered by Majorana in a *lagrangian framework* (in general, the canonical formalism was adopted), obtained from a least action principle: as shown in a Chapter of the coming Volume [35]. After an interesting preliminary study of the problem of the vibrating string, where Majorana obtained a (classical) Dirac-like equation for a two-component field, he then went on to consider a semiclassical relativistic theory for the electron, within which the Klein-Gordon

and the Dirac equations were deduced starting from a semiclassical Hamilton-Jacobi equation. Subsequently, the field equations and their properties were considered in detail, and the quantization of the (free) Dirac field discussed by means of the standard formalism, with the use of annihilation and creation operators. Then, the electromagnetic interaction was introduced into the Dirac equation, and the superposition of the Dirac and Maxwell fields were studied in a very personal and original way, obtaining the expression for the quantized Hamiltonian of the interacting system after a normal-mode decomposition.

Real (rather than complex) Dirac fields, published by Majorana in his famous paper, (9), on the symmetrical theory of electrons and positrons, were considered in the Quaderni in various places (see Quaderno 3, page 3, in ref.[35]), by two slightly different formalisms, namely, by different decompositions of the field. The introduction of the electromagnetic interaction was performed in a quite characteristic manner, and he then obtained an *explicit* expression for the total angular momentum, carried by the real Dirac field, starting from the Hamiltonian.

Some work, as well, at the basis of Majorana's important paper (7), can be found in the Quaderni under consideration (as it will be seen, e.g., in a Section of ref.[35]). We have already seen above, when analyzing the works published by Majorana, that Majorana in 1932 constructed Dirac-like equations for spin one, three-halves, two, etc. (discovering also the method, later published by Pauli & Fiertz, for writing down a quantum-relativistic equation for a generic spin-value). Indeed, in the Quaderni reproduced here, our author, starting from the usual Dirac equation for a 4-component spinor, obtains *explicit* expressions for the Dirac matrices in the cases, for instance, of 6-component and 16-component spinors. Interestingly enough, at the end of his discussion, Majorana also treats the case of spinors with an *odd* number of components, namely, of a 5-component field.

With regard to quantum electrodynamics too, Majorana dealt with it in a lagrangian and hamiltonian framework, by use of a least action principle. As it is *now* currently done, the electromagnetic field was decomposed in plane-wave operators, and its properties were studied within a *full Lorentz-invariant formalism* by employing group-theoretical arguments. Explicit expressions for the quantized Hamiltonian, the creation and annihilation operators for the photons, as well as the angular momentum operator, were deduced in several different bases, along with the appropriate commutation relations. Even leaving aside, for a moment, the scientific value those results had especially at the time when Majorana got them, such manuscripts bear a certain importance from the historical point of view too: They indicate Majorana's tendency to tackle with topics of that kind, nearer to Heisenberg, Born, Jordan and Klein's, than to Fermi's.

As we were saying, and as already pointed out in previous literature [21], in the Quaderni one can find also various studies, inspired to an Oppenheimer's idea, aimed at describing the electromagnetic field within a Dirac-like formalism. Actually, Majorana was interested in describing the properties of the electromagnetic field in terms of a real wavefunction for the photon (see Quaderno 2, page 101a, and Quaderno 17, page 142), an approach that went well beyond the work of contemporary authors. Other noticeable Majorana's investigations concerned the introduction of an *intrinsic* time delay, regarded as a universal constant, into the expressions for electromagnetic retarded fields (see Quaderno 5, page 65); or studies on the modification of Maxwell's equations in the presence of magnetic monopoles (see Quaderno 3, page 163).

Besides purely theoretical work in quantum electrodynamics, some applications as well were carefully investigated by Majorana. This is the case of the free electron scattering (investigated in Quaderno 17, page 133, as reported in ref.[35]), where Majorana gave an *explicit* expression for the transition probability, and the coherent scattering, of bound electrons (see Quaderno 17, page 142b). Several other scattering processes were also analyzed (e.g., in a Chapter of ref.[35]) within the framework of perturbation theory, by the adoption of Dirac's or of Born's method.

As remarked above, the contribution by Majorana in nuclear physics, which became most known to the scientific community of his times, is his theory in which nuclei are formed by protons and neutrons, bound by an exchange-force of a particular kind (which corrected Heisenberg's model). In the Quaderni (cf., e.g., a Chapter of ref.[35]), several pages were devoted to analyze possible forms of the nucleon potential inside a given nucleus, determining the interaction between neutrons and protons. Although general nuclei were often taken into consideration, particular care was given by the author to light nuclei (deuteron, α-particle, etc.). As it will be clearer from what will appear published in ref.[35], the studies performed by Majorana were, at the same time, preliminary studies, and generalizations of what published by him in his well-known publication (8), thus revealing a *very rich* and personal way of thinking. Notice also that, before having understood and thought all what led him to the mentioned paper (8), Majorana had seriously attempted at constructing a *relativistic field theory for nuclei* as composed of scalar particles (see Quaderno 2, page 86), arriving at a characteristic description of the transitions between different nuclei.

Other topics in nuclear physics were broached by our author (and were present in the *Volumetti* too): We shall only mention, here, the study of the energy loss of β particles when passing through a medium, when he deduced the Thomson formula by classical

arguments. Such a work too might a priori be of interest for a correct historical reconstruction, when confronted with the very important theory on nuclear β decay elaborated by Fermi in 1934.

The largest part of the Quaderni is devoted, however, to atomic physics (see, e.g., ref.[35]), in agreement with the circumstance that it was the main research topic tackled by the Fermi group in Rome in the years 1928-1933. Indeed, also the articles published by Majorana in those years deal with such a subject; and echoes of those publications can be found, of course, in the Quaderni: Showing that, especially in the case of the article (5) on the incomplete P' triplets, some *interesting* material did not appear in the published papers.

Several expressions for the wavefunctions and the different energy levels of two-electron atoms (and, in particular, of helium) were discovered by Majorana, mainly in the framework of a variational method aimed at solving the relevant Schrödinger equation. Numerical values for the corresponding energy terms were normally summarized by Majorana in large tables. Some approximate expressions were also obtained by him for three-electron atoms (and, in particular, for lithium), and for alkali; including the effect of polarization forces in hydrogen-like atoms.

In the Quaderni, the problem of the hyperfine structure of the energy spectra of complex atoms was moreover investigated in some detail, revealing[35] the careful attention paid by Majorana to the existing literature. The generalization, for a *non-coulombian* atomic field, of the Landè formula for the hyperfine splitting was also performed by Majorana, together with a *relativistic* formula for the Rydberg corrections of the hyperfine structures. Such a detailed study developed by Majorana constituted the basis of what discussed by Fermi and Segrè in a well-known 1933 paper of theirs on this topic, as acknowledged by those authors themselves.

A small part of the Quaderni was devoted to various problems of molecular physics (see ref.[35]). Majorana studied in some detail, for example, the helium molecule, and, then, considered the general theory of the vibrational modes in molecules, with particular reference to the molecule of acetylene, C_2H_2 (which possesses peculiar geometric properties).

Rather important are some other pages (see Quaderno 8, page 14 and page 46; and Quaderno 6, page 8), where the author considered the problem of ferromagnetism in the framework of Heisenberg's model with exchange interactions. However, Majorana's approach in this study was, as always, *original*, since it did follow neither Heisenberg's, nor the subsequent van Vleck's formulation in terms of a spin Hamiltonian. By using

statistical arguments, instead, Majorana evaluated the magnetization (with respect to the saturation value) of the ferromagnetic system when an external magnetic field acts on it, and the phenomenon of spontaneous magnetization. Several examples of ferromagnetic materials, with different geometries, were by him analyzed as well.

A number of other interesting questions, even dealing with topics that Majorana had encountered during his academic studies at Rome University (as witnessed by a couple of Chapters of ref.[35]), can be found in the Quaderni. This is the case, for example, of the electromagnetic and *electrostatic* mass of the electron (a problem that was considered by Fermi in one of his 1924 known papers); or of his studies on tensor calculus, following his teacher Levi-Civita. Here, we cannot go on in their discussion, however, our aim being that of drawing the attention of the reader to a few specific points only. It is left to the reader's patience the discovery of the large number of exceedingly interesting and important studies that were faced by Majorana, and written by him in the Quaderni [35].

APPENDIX

8. Appendix: About the format of Majorana's scientific manuscripts

As it is clear from what discussed above, Majorana used to put on paper the results of his studies following different behaviors, depending on his opinion about the value of the results themselves. The method used by Majorana for composing his written notes was sometimes the following. When he was investigating a certain subject, he reported his results only in a *Quaderno*. Subsequently, if, after further research on that topic, Majorana reached a simpler and conclusive (in his opinion) result, he then reported the final details also in a *Volumetto*. Therefore, in his preliminary notes we find basically mere calculations, and only in some rare cases an elaborate text, explaining his calculations, can be found in the *Quaderni*. In other words, a clear verbal exposition of the topic can sometimes be found only in the *Volumetti*.

The eighteen Quaderni deposited at the Domus Galilaeana in Pisa are booklets approximately of 15cm$\times$21cm, endowed with a black cover and a red external boundary, as it was common in Italy before the Second World War. Each booklet is composed of about 200 pages, for a total of about 2800 pages. Rarely, some pages were teared off (by Majorana himself), while blank pages in each Quaderno are often present. In few booklets, extra pages written by the author were put in.

An original numbering of the pages is present only in *Quaderno* 1 (on the central top of each page). However, all the Quaderni present a non original numbering (written in red ink) at the left top corner of their odd pages. Blank pages too have been always numbered. Interestingly enough, even if an original numbering by Majorana in general is not present, nevertheless sometimes in a Quaderno it appears an original reference to some pages of that same booklet. Some other strange cross-references, not easily understandable for us, do appear in various booklets.

As it was evident also from a previous catalog of the unpublished manuscripts, prepared long ago by Baldo, Mignani and Recami [14], often in the Quaderni the material regarding the same subject was not written in a sequential, logical order: In some cases, it even appears in the reverse order.

The major part of the Quaderni contains calculations without any explicit explanations; even if, in few cases, an elaborate text is fortunately present.

At variance with what happens for the *Volumetti*, in the Quaderni no dates appear, except for the *Quaderni* 16 (of 1929-30), 17 (started on 20 June 1932) and, probably, 7 (approximately of 1928). Therefore, the actual dates of composition of the manuscripts may be inferred only from a detailed comparison of the topics, studied therein, with what is present in the *Volumetti* and in the published literature, including Majorana's published papers. Some additional information comes from some cross-references explicitly penned by the author himself, referring either to his *Quaderni* or to his *Volumetti*. In few cases, references to the some of the existing literature are explicitly introduced by Majorana.

No consequential or time order is present in the Quaderni, but nevertheless we have been able in ref.[35] to group the topics into four (large) Parts. In any case, in Volume II it will be reported, in a second Index, the complete list of the subjects present in the eighteen Quaderni.

We have done a major effort in carefully checking and typing all equations and tables; and, even more, in writing down a brief presentation of the argument exploited in each subsection. In addition, we have inserted among Majorana's calculations a minimum number of words, when he had left his formalism without any text, trying to facilitate the reading of Majorana's research notebooks, but limiting as much as possible the insertion of any personal comments of ours. Our hope is, by ref.[35], to render the intellectual treasure, contained in the *Quaderni*, accessible for the first time to the widest audience.

At the end of this paper, we attach a short Bibliography. Far from being exhaustive, it just provides references about the topics touched upon in the present article.

9. Acknowledgements

This work has been done in collaboration with A. van der Merwe and R. Battiston. We are moreover indebted, for their kind help, to C. Segnini, the former curator of the Domus Galileana in Pisa, and to the previous curators and directors too. Thanks are moreover due to A. Drago, A. De Gregorio, E. Giannetto, E. Majorana jr., and F. Majorana for valuable cooperation along the years.

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[2] B. Pontecorvo, *Fermi e la fisica moderna* (Editori Riuniti; Rome, 1972); and in *Proceedings of the International Conference on the History of Particle Physics, Paris, July 1982* in *Physique* **43** (1982);

[3] E. Segré, *Enrico Fermi, Physicist* (University of Chicago Press; Chicago, 1970); *A Mind Always in Motion* (University of California Press; Berkeley, 1993).

Accurate biographical information, completed by the reproduction of very many documents, has to be found in the following book (where almost all the relevant documents existing by 2002 —discovered or collected by that author— appeared for the first time):

[4] E. Recami, *Il caso Majorana: Epistolario, Documenti, Testimonianze*, 2nd ed. (Mondadori; Milan, 1991); 4th ed. (Di Renzo; Rome, 2002), pp.1-273.

See also:

[5] E. Recami, "Ricordo di Ettore Majorana a sessant'anni dalla sua scomparsa: l'opera scientifica edita e inedita," in *Quaderni di Storia della Fisica (della Società Italiana di Fisica)* **5** (1999) 19-68, arXive: physics/9810023, and refs. therein. Cf. also: E.Recami, "New Evidences about the Disappearance of the Physicist Ettore Majorana", in *Scientia* **110** (1975) 577-598; and in *Tachyons, Monopoles, and Related Topics*, ed. by E.Recami (North-Holland; Amsterdam, 1978), the Appendix, pp.267-277; as well as the recent bilingual e-print arXiv:0708.2855[physics.hist-ph].

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[10] "Ettore Majorana – Relativistic theory of particles with arbitrary intrinsic angular momentum [Nuovo Cimento **9** (1932) 335]": English translation by C.A. Orzalesi, *Tecnical Report no.792, Feb. 1968 (Dept. of Phys. and Astr.; Univ. of Maryland; College Park).*

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[13] *Ettore Majorana – Scientific Papers*, edited by G.F. Bassani (S.I.F., Bologna, and Springer, Berlin; 2006).

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[15] S. Esposito, E. Majorana Jr., A. van der Merwe and E. Recami (editors), *Ettore Majorana – Notes on Theoretical Physics* (Kluwer Acad. Pub.; Dordrecht and Boston, 2003).

The original Italian version, instead, has been successively published in:

[16] S. Esposito and E. Recami (editors), *Ettore Majorana – Appunti Inediti di Fisica Teorica* (Zanichelli; Bologna, 2006).

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[17] *Ettore Majorana – Lezioni all'Università di Napoli*, edited by B. Preziosi (Bibliopolis; Neaples, 1987);

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[18] *Ettore Majorana – Lezioni di Fisica Teorica*, edited by S. Esposito (Bibliopolis; Neaples, 2006).

See also:

[19] A. Drago and S. Esposito, "Ettore Majorana's course on theoretical physics: A recent discovery," in *Phys. Persp.*, in press [e-print physics/0503084].

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[23] E. Majorana, "Ricerca di un'espressione generale delle correzioni di Rydberg, valevole per atomi neutri o ionizzati positivamente" (Search of a general expression for Rydberg's correction, valid for neutral or positively charges atoms), *Nuovo Cimento* **6** (1929) 14-16. The corresponding original material is all contained in Refs.[15,16], while a comment is in S. Esposito, "Again on Majorana and the Thomas-Fermi model: A comment about physics/0511222," [arXiv:physics/0512259];

[24] S. Esposito, "A peculiar lecture by Ettore Majorana," *Eur. J. Phys.* **27** (2006) 1147; "Majorana and the path-integral approach to Quantum Mechanics," *Ann. Fond. Louis de Broglie* **31** (2006) 1;

[25] S. Esposito, "Hole theory and Quantum Electrodynamics in an unknown manuscript in French by Ettore Majorana," *Found. Phys.*, in press [arXiv:physics/0609137].

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[33] E. Di Grezia and S. Esposito, "Majorana and the quasi-stationary states in nuclear physics" [arXiv:physics/0702179];

[34] A. Drago and S. Esposito, "Following Weyl on quantum mechanics: The contribution of Ettore Majorana," *Found. Phys.* **34** (2004) 871.

Last but not least, the English translation of a selection of the *Quaderni* is going to be published in:

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